BINOD BIHARI MAHTO KOYALANCHAL UNIVERSITY, DHANBAD

SYNOPSIS

ON

Study of Some Problems on Electrically Conducting Fluid Flows in Presence of Magnetic Field: An Analytical approach

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SYNOPSIS

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1. STATE OF ART

Magnetohydrodynamics or simply MHD in its abbreviated form is the phenomenon of motion of electrically conducting fluid in the presence of a magnetic field. It may be observed with liquid metals, electrolytes, salt water, two phase mixtures, plasmas etc. The term magnetohydrodynamics is the combination of three words 'magneto' meaning magnetic field, 'hydro' meaning water (or liquid) and 'dynamics' meaning movement. Magnetofluiddynamics and hydromagnetics are less frequently used short forms of MHD. Actually, the union of three separate disciplines, namely, fluid dynamics, electrodynamics and thermodynamics forms the basis of MHD. It can be said that MHD is the study of interaction between the magnetic fields and moving conducting fluids. When a conducting fluid moves through a magnetic field, an electric field and consequently a current may be induced and, in turn, the current interacts with the magnetic field to produce a body force on the fluid.

The interaction between the magnetic field and motion of the fluid arises partially as a result of laws of Faraday and Ampere, and partially because of Lorentz force experienced by a current carrying body. The process can be split into three ways-

- The relative movement of a conducting fluid and a magnetic field causes an e.m.f to develop in accordance with the Faraday's law of induction.
- The induced currents must, according to ampere's law, give rise to a second induced magnetic field. This adds to the original magnetic field and the change is such that the fluid appears to drag the magnetic field lines along it.
- The combined magnetic field (imposed plus induced) interacts with the induced current density to give rise to Lorenz force. This acts on the conductor and is generally directed so as to inhibit the relative movement of the magnetic field and the fluid.

The consequence of the last two effects is basically the same where relative motion of the fluid and the field is reduced. Fluids can drag magnetic field lines and in turn magnetic field drags conducting fluids. The fluid and the magnetic field appear to be 'frozen together', which is the essence of magnetohydrodynamics. MHD treats, in particular, conducting fluids, whether liquid or gaseous, in which certain simplifying postulates are made. These, are generally, that the Maxwell displacement current is neglected, and the fluid may be treated as a continuum, without mean-free path effect. A large variety and ubiquitous nature of magnetic field shows that MFD phenomena must occur where conducting fluids

are available. Hence MHD applies quite well to astrophysics since over 90% of matter content of universe is made up of plasma, including stars, the interplanetary medium (space between planets), interstellar medium (space between the stars), nebulae and jets. MHD is also related to engineering problem such as plasma confinement, liquid metal cooling of nuclear reactors and electromagnetic casting among others.

2. PROBLEM TO BE INVESTIGATED

The proposed research work for Ph.D. is as follows:

(i) Analytical solution of steady/unsteady second grade MHD fluid flow

(ii) Exact solution of electrically conducting fluid with Hall current

(iii) Analytical study of MHD fluid flow through porous media to find exact solution

(iv) Exact solution of MHD fluid flow in rotating frame of reference

3. OBJECTIVE OF THE INVESTIGATION

The objective of the present research work is as follows:

(i) To obtain Analytical solution using transformation techniques such as Hodograph/ Magnetograph transformation

(ii) To find exact solution using Martin's method based on differential geometry approach

(iii) To use Inverse method for finding exact solution

(iv) To analyze the flow pattern by studying the parameters which affect the fluid flow

4. MOTIVATION

The first investigations in MFD were due to Faraday in 1836 [1] when he attempted to detect the electric field induced by motion of conducting water of Thames river in presence of earth's magnetic field. But he failed in his experiment due to low electrical conductivity of the fluid. However, H. Alfven was responsible for advancement of MHD as in 1942 he established transverse waves in electrically conducting fluids and published some papers on the sunspot theory. In 1950 he published a book called "Cosmical Electrodynamics" [2] which was the first correlation of advances in MHD.

The mathematical study of MHD is concerned primarily with partial differential equations which arise from well-known physical conservation laws. Because of complexity of governing equations most of the MHD investigations till date have been carried out with some simplifying assumptions. The governing equations of MHD fluid flows form systems of non-linear partial differential equations which are very difficult to solve for exact solutions.

Ames in 1965 [3] gave excellent methods employed for solving these equations and their applications. However, the search for the solutions of these nonlinear systems continues. The two definitions for an exact solution [1] of an ordinary viscous incompressible flow problem are:

(i) A set of functions for the velocity components and the fluid pressure constitutes an exact solution of the flow equations if these equations are satisfied by these functions for all values of independent variables and for all values of the fluid density and the fluid viscosity.

(ii) A set of functions for the velocity components and the fluid pressure constitutes an exact solution or exact integral of the flow equations if these equations and the boundary conditions of a realistically imposed physical problem are satisfied for all values of the independent variables and for all values of fluid density and the fluid viscosity.

The exact solutions as reported by [4] are very important as they are not only solutions of some fundamental flows but also due to the uniform validity of exact solutions, the phenomena described by the flow equations can be more closely studied. Also, they serve as accuracy checks for many approximate methods, whether experimental, numerical or asymptotic. Our motivation lies in finding the exact solution of some MHD fluid flows analytically which will provide nature of permissible solutions of the system. The solutions thus obtained will also contribute significantly towards increasing the knowledge of MHD flows and lead to future researchers to its application for real problems.

5. RESEARCH DESIGN AND METHODOLOGY

In our proposed research work we will use the basic MHD equations (in accordance with the chosen problem) which are inherently non-linear partial differential equations and have no general solution. These equations then will be converted to suitable forms using various transformation techniques and methods such as Hodograph / Magnetograph transformations, Inverse method, use of Martin's method based on differential geometry etc. We will transform the governing equations in solvable forms and use these equations to find the exact solutions and get the information about the flow structure.

5.1 MHD equations:

When the fluid velocity is small compared to the velocity of light, the displacement current is negligible. Also, fluids are almost neutral; charge density of fluid and convection current being very small compared to conduction current, can be neglected. The basic MHD equations for the flow of homogenous, isotropic, electrically conducting, viscous, incompressible fluid of constant density ρ, constant electrical conductivity σ, and constant kinematic coefficient of viscosity η in a rotating frame with uniform angular velocity $\vec{\Omega}$ through porous media having permeability of medium k are as follows:

Maxwell's equations-

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t},$$

$$\vec{\nabla} \times \vec{B} = \mu \vec{J},$$

$$\vec{\nabla} \cdot \vec{B} = 0,$$

and

$$\vec{\nabla} \cdot \vec{E} = 0,$$

The constitutive field relation

$$\vec{B} = \mu \vec{H},$$

Generalized Ohm's law for a moving conductor-

$$\vec{J} = \sigma[\vec{E} + \vec{V} \times \vec{B}],$$

Ohm's law for moving conductor taking Hall current into account

$$\vec{J} + \frac{\omega_e \tau_e}{B_o}(\vec{J} \times \vec{B}) = \sigma[\vec{E} + \vec{V} \times \vec{B}],$$

Equation of continuity-

$$\vec{\nabla} \cdot \vec{V} = 0,$$

Conservation of momentum-

$$\rho \frac{\partial \vec{V}}{\partial t} + \rho[(\vec{V} \cdot \vec{\nabla})\vec{V} + 2\vec{\Omega} \times \vec{V}] = \rho \vec{F} - \vec{\nabla} P + \mu \vec{J} \times \vec{H} + \eta \nabla^2 \vec{V} - \frac{\eta}{k} \vec{V},$$

Conservation of energy-

$$\rho T \frac{\partial}{\partial t} + \rho T(\vec{V} \cdot \vec{\nabla}) = \phi^* + \frac{Q^2}{\sigma} + \vec{\nabla} \cdot (K^* \vec{\nabla} T),$$

where,

$\vec{V}$ = fluid velocity vector, $\vec{B}$ = magnetic induction vector, $\vec{J}$ =current density, $\vec{E}$ =electric field, $\vec{H}$ =magnetic field, $\vec{F}$ =body force per unit mass including gravitational force, P = fluid pressure, T = absolute temperature of fluid, K^* =coefficient of heat conductivity, ϕ^* =viscous dissipation, μ =magnetic permeability, ω_e =cyclotron frequency, τ_e =electron collision time, B_o = applied magnetic field, S = entropy of fluid, $|\vec{J}| = Q$, P' =the reduced pressure given by $P - \frac{1}{2}\rho|\vec{\Omega} \times \vec{r}|$, $\vec{r}$ being the position vector from the origin to general point at which Eulerian fluid velocity $\vec{V}$ is measured relative to a rotating frame [5].

5.2 Hodograph transformation

Hodograph transformation is a technique used to transform non-linear partial differential equations into linear version. It consists of interchanging the dependent and independent variables in the equation to achieve linearity. In partial differential equations, if the independent variables and dependent variables are switched, then the space of independent variables is called hodograph space (in two dimensions, the hodograph plane). A curve in which the radius vector represents the velocity of a moving particle (i.e., the plane of the velocity vector $\vec{V}$) is called hodograph plane. Ames [3] has given an excellent survey of the method. Many authors including O.P. Chandna et al. [6] [7] [8] [9] [10] [11] have applied Hodograph and Legendre transform to investigate steady plane viscous flows, non-Newtonian flows and constantly inclined, aligned, transverse and orthogonal MHD non-Newtonian flows. C. Thakur and Ram Babu Mishra [12] used hodograph transformation technique to obtain the exact solution in case of constantly inclined two phase MFD method for non-Newtonian transverse flow. Sayantan Sil and Manoj Kumar and S. Singh [13] applied hodograph transformation to constantly inclined two phase rotating MHD flow through porous media. Also, Manoj Kumar, Sayantan Sil and Mantu Prajapati [14] used Hodograph transformation and found some exact solution for rotating MHD flow. Recently Sayantan Sil and Manoj Kumar [15] used this transformation to find exact solution of plane micropolar fluid through porous medium.

5.3 Magnetograph transformation

If the magnetic field vectors of an MHD fluid are laid off from a fixed point, the extremities of these vectors trace out curves, called magnetograph. Magnetograph transformation is analogous to the Hodograph transformation and obtains an equivalent linear system by interchanging the roles of dependent and independent variables. When transformations are

employed to interchange the roles of the two independent variables and the two components of the magnetic field in the physical plane, these transformations are the magnetograph transformations. S. N. Singh [16] applied magnetograph transformation in MFD flows. C. S. Bagewadi and Sidabasappa [17] studied variably inclined rotating MHD flows in magnetograph plane. C. Thakur and B. Singh [18] studied variably inclined MHD flow using in magnetograph plane. Also Y. Venkateshappa, Siddabasappa and R. Basavanna [19]found solutions for steady flow in a rotating reference frame of viscous fluids with finite electrical conductivity by employing magnetograph transformation method.

5.4 Inverse method

The inverse method is another method for solving the non-linear partial differential equations. This method consists of making certain hypothesis a priori on the form of the velocity field and pressure. The inverse method [20] reduces a system of differential equations in three independent variables to a system having two or one independent variable which may lead to an exact solution. Generally, in inverse method solution is sought by the assumption of vorticity being proportional to the stream function of the flow. J. B. Jeffery [21] in 1915 used inverse method in two-dimensional steady motion of a viscous fluid. P. F.Nemenyi [20] , in his book in 1951 gave a detailed survey on inverse method. Many authors [22] [23] [24] solved Navier-Stokes equations and second grade steady and unsteady fluid flow using this method. O. P. Chandna and E. O. Oku-Ukpong[25][26] also used this method in MHD and non-MHD fluid flow study. F. Labropulu [27] [28] [29] also used this method to study different flow problems. Muhammad R Mohyuddin and Ehsaan Elaahi Asraf [30] studied the inverse solution of second grade fluid for porous medium channel and hall current effects. Manoj Kumar [31] found exact solution of Non-Newtonian fluid with Hall effect having prescribed vorticity distribution function using inverse method. Manoj Kumar and Sayantan Sil along with R. Sharma and C. Thakur [32][33] [34] used inverse method to find exact solutions of MHD flow taking different forms of vorticity function proportional to stream function. More recently Mantu Prajapati, Sayantan Sil and Manoj Kumar [35] extended the work of MHD flow through porous media with variable permeability and found some solution using inverse method.

5.5 Martin's method

Martin [36] introduced a new method to study two-dimensional steady plane flows of viscous fluid in the absence of external forces. In this approach a curvilinear coordinate system (ϕ, ψ) in the plane of flow is introduced and the coordinate lines $\phi =$ constant are taken as streamlines of the flow while the coordinate lines $\psi =$ constant are arbitrary. As mentioned by R. K. Naeem and M. Jamil [37] Martin discussed some flows and suggested fifteen different types of flows, which could be studied using this new method. Govindaraju [38] further exploited this method for viscous fluid flow. Nath and Chandna [39] also used this method to study viscous MHD fluid flow. Also, Chandna and Garg [40] and Chandna, Barron and Garg [41] applied the Martin's approach for MHD flow problems. Bagewadi and

Siddabassappa [42] studied plane rotating viscous MHD flows employing Martin's method. O.P. Chandna and F. Labropulu [43] employed the results of differential geometry to obtain the exact solution of steady plane flow. Also, C. Thakur, Manoj Kumar and M. K. Mahan [44] applied Martin's method to discuss the geometry of constantly inclined MHD flow through porous media. Manoj Kumar and Sayantan Sil [45] applied Martin's method to find the exact solution of steady plane rotating aligned MHD flows. Also recently, Sayantan Sil, Mantu Prajapati and Manoj Kumar [46] used Martin's method to find a class of exact solution of MHD fluid flows.

6. WORK PLAN

The governing equations of MHD flow are non-linear partial differential equations and have no general solution. Only a small number of exact solutions have been found because the non-linear terms do not vanish automatically. Our main aim is to use the various methods/techniques to transform the governing equations in suitable forms and use these transformed equations to find the exact solutions and get the information about the flow structure.

7. TOOLS TO BE USED IN THE COLLECTION OF DATA

The following methods will be used for the collection of data of existing knowledge in the field of our proposed research:

(i) From suitable text books

(ii) From Research Journals

(iii) From other published Ph.D. theses

(iv) From appropriate websites and content available in internet

8. SIGNIFICANCE OF THE STUDY

In this proposed research work we shall try to find out exact solutions of some problems involving MHD fluid flows. As described by Wang [4] the exact solutions are very important as they represent fundamental flows and helps in understanding the basic phenomena described by the fluid flow equations. Also, the exact solutions help in checking the accuracies of approximate methods, which may be experimental, numerical or asymptotic. The advances in computer technology makes it possible to determine a complete numerical integration but the accuracy of the result can only be validated by comparison with the exact solution. This study will also help in getting information about the flow structure and will increase knowledge regarding MHD.

9. RELEVANCE OF THE RESEARCH TO PRESENT DAY PROBLEMS AND NEED OF THE SOCIETY

We shall find the exact solution of the flow problem in which physical boundary value problem is not considered. But the results thus obtained can be used to solve some real physical problems. Also, these solutions will provide nature of permissible solutions to the system. Also, this research work will increase knowledge about the flow of MHD in general and will have an impact on scientific and real-life applications like enhanced cooling and heating of fluids, to enhance heat exchanging process, to create power from two phase mixtures, in aerospace engineering to modify aerodynamic forces, etc. [47]

10. CONTRIBUTION TO THE EXISTING KNOWLEDGE

Our proposed work will help immensely in getting information regarding the flow structure and also enhance knowledge of MHD. The exact solutions found will lead future researchers for application in a real problem. The real boundary value problems of fluid mechanics are exceedingly difficult and progress for their study and solution would be made easy by the rigorous mathematical approach of this analytical study.

11. FUTURE WORK

Most of the theoretical analysis in MHD is governed by complicated non-linear partial differential equations. It is very difficult to find the exact solutions as there is no existing general theory for completely solving these non-linear partial differential equations. Our work will provide an opening for some new approach to solve such problems, some of them are already taken up and works are in progress. We do not make any attempt to solve any boundary value problems but in future by using the approach of this research work and the exact solutions thus obtained; work towards boundary value problems of practical importance can be pursued. In future, researchers may use our solution techniques and results in problems of practical importance such as MHD power generator, cooling of nuclear reactor etc. and may also apply some boundary value conditions to gather appropriate information.

12. TENTATIVE CHAPTERIZATIONS

Tentative chaterizations of the proposed research work are as follows :

Ch.1. Introduction of MHD.

Ch.2. Analytical solution of MHD fluid flow using Hodograph/Magnetograph transformation method.

Ch.3. Exact solution of MHD fluid using Inverse method.

Ch.4. Application of Martin's method for the solution of MHD fluid flow.

Ch.5. Conclusion.

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